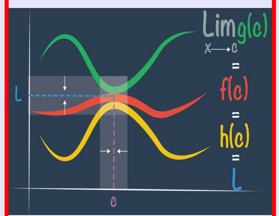
Calculus I

Lecture 38



Feb 19-8:47 AM

Verify Conditions of Rolle's thrm for $f(x)=\cos 2x$ on $\left[\frac{\pi}{8},\frac{7\pi}{8}\right]$, then find all numbers that satisfy the conclusion of Rolle's thrm.

$f(x)=\cos 2x$

Cont. $(-\infty,\infty)$

diff. $(-\infty,\infty)$

$f\left(\frac{\pi}{8}\right)=\cos\frac{2\pi}{8}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$f\left(\frac{7\pi}{8}\right)=\cos\frac{2\cdot 7\pi}{8}=\cos\frac{7\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ $\Rightarrow f(a)=f(b)$

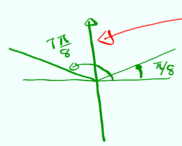
There is at least a number C in $\left(\frac{\pi}{8},\frac{7\pi}{8}\right)$ such that $f'(c)=0$

$f(x)=\cos 2x$ $\quad -2\sin 2x=0$

$f'(x)=-2\sin 2x$

$2x=0+k\cdot 2\pi \rightarrow x=k\pi$

$2x=\pi+k\cdot 2\pi \rightarrow x=\frac{\pi}{2}+k\pi$



$k=0$ $\quad x=0,\ \boxed{\frac{\pi}{2}}$

$k=1$ $\quad x=\pi,\ \frac{3\pi}{2}$

$k=2$ $\quad x=2\pi,\ \frac{5\pi}{2}$

$C=\frac{\pi}{2}$ is in $\left(\frac{\pi}{8},\frac{7\pi}{8}\right)$

Nov 6-7:26 AM

Verify the conditions of MVT for $f(x) = x^3 - 3x + 2$ on $[-2,2]$, then find all numbers c in $(-2,2)$ that satisfy the conclusion of MVT.

$f(x)$ cont $[-2,2]$

$f(x)$ diff $(-2,2)$

$f(x)$ is a polynomial functions $\Rightarrow$ Cont. & diff $(-\infty,\infty)$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$3c^2 - 3 = \frac{4-0}{2-(-2)}$$

$f'(x) = 3x^2 - 3$

$3c^2 - 3 = 1$

$f(2) = 2^3 - 3(2) + 2 = 4$

$c^2 = \frac{4}{3}$ $\quad c = \pm\frac{2}{\sqrt{3}}$

$f(-2) = (-2)^3 - 3(-2) + 2 = 0$

$\boxed{c = \pm\frac{2\sqrt{3}}{3}}$

$c \approx \pm 1.2$

$(-2,2)$

Nov 6-7:35 AM

Suppose $3 \le f'(x) \le 5$ for all values of x

show $18 \le f(8) - f(2) \le 30$.

Since $f(x)$ is diff. for all values of x, then it is cont.

By MVT on $[2,8]$, $f'(c) = \dfrac{f(8)-f(2)}{8-2}$

Since $3 \le f'(x) \le 5$, $\quad 3 \le \dfrac{f(8)-f(2)}{8-2} \le 5$

Multiply by 6 $\quad \boxed{18 \le f(8) - f(2) \le 30}$

Nov 6-7:41 AM

$f(x) = \dfrac{x^2}{x-1}$

Domain $(-\infty, 1) \cup (1, \infty)$ VA $x = 1$

Y-Int $(0,0)$, X-Int $(0,0)$ twice(even)

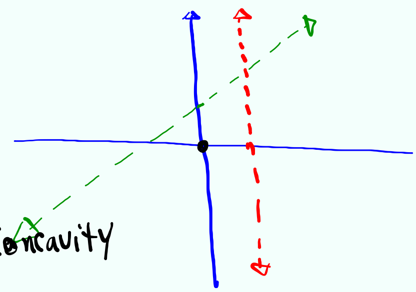
Long Division

$$\begin{array}{r} x + 1 \\ x-1 \enclose{longdiv}{x^2 + 0x + 0} \\ -(x^2 - x) \\ \hline x + 0 \\ -(x - 1) \\ \hline 1 \end{array}$$

$f(x) = x + 1 + \dfrac{1}{x-1}$

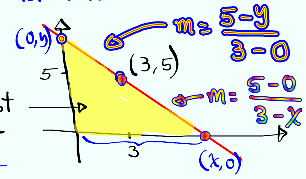
$y = x + 1$ is slant Asymptote

Find
$f'(x)$, C.P.
$f''(x)$, P.I.P.
Do Sign Chart
Discuss inc., Dec., Concavity
Graph



Nov 6-7:46 AM

Find equation of a line that contains the point $(3,5)$ and cuts off Smallest area in QI.



Area $= \dfrac{xy}{2}$

Minimum

$\dfrac{5-y}{3} = \dfrac{5}{3-x}$

$5 - y = \dfrac{15}{3-x}$

$y = 5 - \dfrac{15}{3-x}$

$y = \dfrac{5(3-x) - 15}{3-x}$

$y = \dfrac{-5x}{3-x}$

$y = \dfrac{5x}{x-3}$

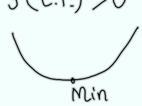
Area $= \dfrac{xy}{2} = \dfrac{x}{2}\left(\dfrac{5x}{x-3}\right)$

$f(x) = \dfrac{5x^2}{2(x-3)}$

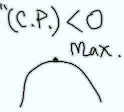
Find $f'(x)$, Find C.P.

Find $f''(x)$, evaluate $f''(\text{C.P.})$

$f''(\text{C.P.}) > 0$



$f''(\text{C.P.}) < 0$



Nov 6-7:52 AM

Prove if $f(x)$ is diff. at $x=a$, then $f(x)$ is Cont. at $x=a$.

Given $f(x)$ is diff. at a $\to f'(a)=\lim\limits_{x\to a}\frac{f(x)-f(a)}{x-a}$

Show $f(x)$ is Cont. at a $\to \lim\limits_{x\to a} f(x)=f(a)$

$$f(x)=f(x)-f(a)+f(a)$$

$$f(x)=\frac{f(x)-f(a)}{x-a}\cdot(x-a)+f(a)$$

$$\lim_{x\to a} f(x)=\boxed{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}\cdot\lim_{x\to a}(x-a)+\lim_{x\to a} f(a)$$

$$\lim_{x\to a} f(x)=f'(a)\cdot(a-a)+f(a)$$

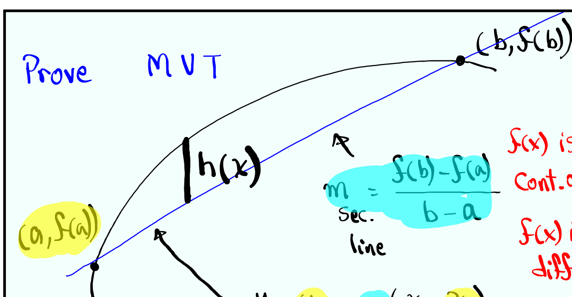
$$=f'(a)\cdot 0+f(a)$$

$$\lim_{x\to a} f(x)=f(a)$$

$f(x)$ is Cont. at $x=a$.

Nov 6-8:03 AM

Prove MVT



$m_{\text{sec. line}}=\frac{f(b)-f(a)}{b-a}$

$f(x)$ is Cont. on $[a,b]$

$f(x)$ is diff. on (a,b)

$$y-y_1=m(x-x_1)$$

$$y-f(a)=\frac{f(b)-f(a)}{b-a}(x-a)$$

$$\boxed{y=\frac{f(b)-f(a)}{b-a}(x-a)+f(a)}$$

$h(x)$ is vertical distance between Curve & line

$$h(x)=f(x)-\frac{f(b)-f(a)}{b-a}(x-a)-f(a)$$

$h(x)$ is Cont. on $[a,b]$ ✓

$h(x)$ is diff. on (a,b) ✓

$h(a)=h(b)$ ✓

Now we can apply Rolle's thrm

$h'(c)=0$

Nov 6-8:10 AM

$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a} \cdot 1 - 0$$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

By Rolle's thrm $h'(c) = 0$

$$f'(c) - \frac{f(b)-f(a)}{b-a} = 0$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Conclusion of MVT

Nov 6-8:19 AM

$f''(x) = \sin x + \cos x \qquad f(0)=3, \; f'(0)=4$

find $f(x)$

$$f'(x) = -\cos x + \sin x + C$$

$$f'(0) = -\cancel{\cos 0}^{\,1} + \cancel{\sin 0}^{\,0} + C = 4$$

$$\boxed{C=5}$$

$$f'(x) = -\cos x + \sin x + 5$$

$$f(x) = -\sin x - \cos x + 5x + C$$

$$f(0) = -\cancel{\sin 0}^{\,0} - \cancel{\cos 0}^{\,1} + \cancel{5(0)}^{\,0} + C = 3$$

$$\boxed{C=4}$$

$$\boxed{f(x) = -\sin x - \cos x + 5x + 4}$$

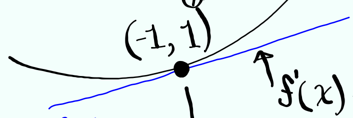
Nov 6-8:22 AM

Find the function $f(x)$ such that

$f'(x) = x^3$ and the line $x+y=0$ is

tan. to the graph of $f(x)$.

$(-1, 1)$



$f'(x) = -1$

$x+y=0$

$y = -x$

slope $= -1$

$f(x) = \frac{1}{4}x^4 + C$

$x^3 = -1$

$x = -1$

$f(-1) = 1$

$\frac{1}{4}(-1)^4 + C = 1$

$\frac{1}{4} + C = 1$

$C = 1 - \frac{1}{4}$

$\boxed{C = \frac{3}{4}}$

$$\boxed{f(x) = \frac{1}{4}x^4 + \frac{3}{4}}$$

Nov 6-8:29 AM